

Pre-Calculus Part 1

Find the solutions of each equation on the interval $[0, 2\pi)$.

1. $\sin\left(\frac{3\pi}{2} + x\right) + \sin\left(\frac{3\pi}{2} + x\right) = -2$

(SHOW WORK)

$$\sin\left(\frac{3\pi}{2} + x\right) + \sin\left(\frac{3\pi}{2} + x\right) = -2$$

$$\sin\left(\frac{5x}{2}\right) + \sin\left(\frac{5x}{2}\right) = -2$$

$$2 \sin\left(\frac{5x}{2}\right) = -2$$

$$\sin\left(\frac{5x}{2}\right) = -1$$

$$\frac{5x}{2} = \frac{-\pi}{2}$$

$$\frac{5x}{2} = 2\pi n - \frac{\pi}{2}, \quad n \in \mathbb{Z}$$

$$x = \frac{2\pi(2n - \frac{1}{2})}{5}, \quad n \in \mathbb{Z}$$

2. The following table shows the estimated populations and annual growth rates for four countries in the year 2000. Find the expected population of each country in 2025, assuming their annual growth rates remain steady.

Country	Population in 2000	Growth Rate
Australia	19,169,000	0.6%
China	1,261,832,000	0.9%
Mexico	100,350,000	1.8%
Zaire	51,965,000	3.1%

(SHOW WORK)

$$\text{Australia} = 19,169,00 \times 0.006 \times 25 = 22271200$$

$$\text{China} = 1,261,832,000 \times 0.009 \times 25 = 1,580,220,877$$

$$\text{Mexico} = 100,350,000 \times 0.018 \times 25 = 157,380,127$$

$$\text{Zaire} = 51,965,000 \times 0.031 \times 25 = 112,794,819$$

3. Solve

(SHOW WORK)

$$6 + \sqrt[5]{249 - 2x} = 7$$

$$6 + \sqrt[5]{249 - 2x} = 7$$

$$\sqrt[5]{249 - 2x} = 7 - 6$$

$$\sqrt[5]{249 - 2x} = 1$$

$$249 - 2x = \sqrt[5]{1}$$

$$1^5 = 249 - 2x$$

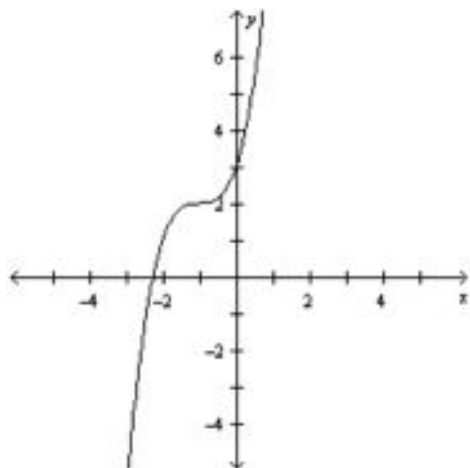
$$249 - 2x = 1$$

$$-2x = 1 - 249$$

$$\frac{-2x}{-2} = \frac{-248}{-2}$$

$$X = 124$$

4. Estimate and classify the critical points for the graph of the function.



Points of function are point $(0, -2\frac{1}{2})$ and point $(0, 3)$.

5. Astronomers classify stars according to their brightness by assigning them a stellar “magnitude.” The *higher* the magnitude the *dimmer* the star. The dimmest stars visible to the naked eye have stellar magnitudes of 6. The table below shows the relative brightness of different stellar magnitudes.

Stellar Magnitude	1	2	3	4	5	6
Relative Brightness	100	40	16	6.3	2.5	1

Stellar magnitude problem

$$\frac{L}{4\pi d^2} = \text{Apparent (Relative) Brightness}$$

a. Find an equation that gives the relative brightness in terms of stellar magnitude.

$$(1/2.512)^m (1/4\pi d^2)$$

b. Use this equation to find the relative brightness of a star with magnitude 9.

(SHOW WORK)

$$(\frac{1}{2} \cdot 512)^9 (\frac{1}{4}\pi (10^2))$$

$$(\frac{1}{2} \cdot 512)^9 (\frac{1}{4}\pi 100)$$

$$(.398)^9 (1/1, 256.637)$$

$$(.398)^9 (.000796)$$

$$(3,225.027) (.000796)$$

$$2.567/100 = .0257$$

6. A restaurant offers a lunch special in which a customer can select from one of the 7 appetizers, one of the 10 entrees, and one of the 6 desserts. How many different lunch specials are possible?

(SHOW WORK)

$$7 + 10 + 6 = 23 \text{ different lunch specials.}$$

7. Divide using long division

$$\frac{-3x^5 + 11x^4 + 33x^3 - 26x^2 - 36x - 6}{x^3 + 6x^2 - 3x - 5}$$

(SHOW WORK)

$$319x^2 + 467x - 756$$

$$-3x^2 + 29x - 150 + \quad x^3 + 6x^2 - 3x - 5$$

$$x^2 + 6x^2 - 3x - 5 \sqrt{-3x^5 + 11x^4 + 33x^3 - 26x^2 - 36x - 6}$$

$$-3x^5 - 18x^4 + 9x^3 - 15x^2$$

$$= 0 \quad 29x^4 + 24x^3 - 11x^2 - 3x$$

$$29x^4 + 174x^3 - 8x^2 - 145x$$

$$0 \quad -150x^3 + 19x^2 + 17x - 6$$

$$-150x^3 - 300x^2 - 450x + 76$$

$$0 \quad 319x^2 + 467 - 756$$

Answer: $-3x^2 + 29x - 150 + \frac{319x^2 + 467x - 750}{x^3 + 6x^2 - 3x - 5}$

8. Condense each expression.

$$5\log_3 x - \frac{1}{4} \log_3 (8 - x)$$

$$\log b x^c = c \log b x$$

$$\log 5 x^5 - \frac{1}{4} \log (8 - x)$$

$$\log 5 x^5 - \log 5 (\sqrt[4]{8 - x})$$

Answer: $\log 5 (x^5 / \sqrt[4]{8 - x})$

9. A gear of radius 6.1 cm turns at 11 revolutions per second. What is the linear velocity of the gear in meters per second?

(SHOW WORK)

$$= 2 \pi r$$

$$= 2(3.14)(6.1) \text{ m/s}$$

$$= 33.3$$

$$11(33.3) =$$

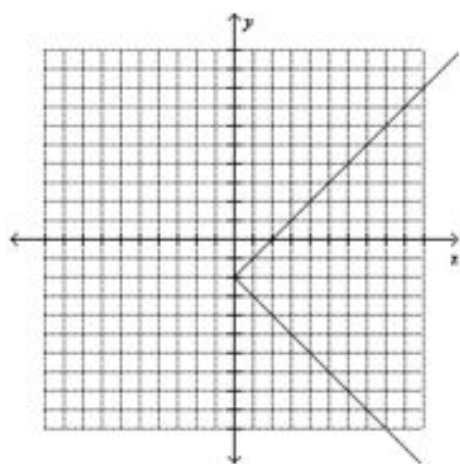
$$LV = 366.3 \text{ m/s}$$

10. Use the graph to determine the domain and range of the relation, and state whether the relation is a function.

Not a function

Domain $(+\infty, -\infty)$

Range: $(0, -2)$



11. Simplify. **(SHOW WORK)**

$$\frac{\tan 9x - \tan 5x}{1 + \tan 9x \tan 5x}$$

$$\tan 9x - \frac{\tan 5x}{1 + \tan 9x \tan 5x}$$

$$9 \tan x - \frac{\tan x}{14 \tan x - 1}$$

$$\frac{(14 \tan x + 1)(9 \tan x) + (-\tan x)}{14 \tan x + 1}$$

$$\left(\frac{9 \tan x + 14 \tan x + 9 \tan x}{14 \tan x + 1} - \tan x \right)$$

$$\frac{(126 + 1 + 14 \tan x + 14 \tan x + 14 \tan x) - \tan x}{14 \tan x + 1} - \tan x$$

$$\frac{126 + 14 \tan x + 14 \tan x + 14 \tan x - \tan x}{14 \tan x + 1}$$

$$\frac{2 \tan x (63 \tan x + 2)}{14 \tan x + 1}$$

$$= \frac{2 \tan x (63 \tan x + 2)}{14 \tan x + 1}$$

Find each $f(c)$ using synthetic substitution.

$$12. f(x) = 5x^5 + 10x^4 + 3x^3 + 8x^2 - 6x - 3; c = 3$$

(SHOW WORK)

$$f(x) = 5x^5 + 10x^4 + 3x^3 + 8x^2 - 6x - 3y \quad c = 3$$

$$f(3) = 5(3)^5 + 10(3)^4 + 3(3)^3 + 8(3)^2 - 6(3) - 3$$

$$f(3) = 5(243) + 10,000 + 72 - 18 - 3$$

$$f(3) = 1,215 + 10,000 + 72 - 18 - 3$$

$$f(3) = 11,266$$

13. Suppose θ is an angle in the standard position whose terminal side is in Quadrant I and . Find the exact values of the five remaining trigonometric functions of θ .

$$\text{Sino} = \frac{84}{85} = \frac{0}{h} = \frac{y}{r}$$

$$85^2 = c^2 + 84^2$$

$$\text{Sino} = \frac{84}{85}$$

$$7,225 = c^2 + 7,056$$

$$1. \text{Coso} = \frac{13}{84}$$

$$7,225 - 7,056 = c^2$$

$$2. \text{Tano} = \frac{84}{-13}$$

$$169 = c^2$$

$$3. \text{Csco} = \frac{85}{84}$$

$$\sqrt{169} = \sqrt{c^2}$$

$$4. \text{Seco} = \frac{85}{-13}$$

$$C = 13$$

$$5. \text{Coto} = \frac{-13}{84}$$

14. A truck driver travels at 59 miles per hour. The truck tires have a diameter of 30 inches. What is the angular velocity of the wheels in revolutions per minute (rpm)?

(SHOW WORK)

Circumference of tire = $\pi D = 30$ inches

One revolution of tire = 26π inches

In one mile of tire = 63360

$$59(63360)/30/3600 = 35 \text{ radians/s}$$

$$= 35 \text{ radius/s}$$

15. Determine the zeros for and the end behavior of $f(x) = x(x - 4)(x + 2)^4$.

$$f(x) = x(x - 4)(x + 2)^4$$

$$0 = x(x - 4)(x + 2)^4$$

$$X = 0 \quad x + 2 = 0$$

$$X - 4 = 0 \quad x = -2$$

$$X = 4$$

So $x = 0$, $x = 4$, and

$$X = -2$$

16. Verify . $\sin(180^\circ - \theta) = \sin \theta$

(SHOW WORK)

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\text{And } \cos 180^\circ = -1$$

$$\text{Or } \sin(A - B) = \sin(A) \cos(B) - (\cos(A)) \sin(B)$$

$$\text{So } \sin(180^\circ - 0) = 0 - (-1)\sin(0)$$

$$= \sin(0)$$

17. The hourly temperature at Portland, Oregon, on a particular day is recorded below.

1 A.M.	2	3	4	5	6	7	8	9	10	11	12 Noon
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46°	44°	43°	41°	40°	40°	41°	43°	46°	52°	65°	69°
1 P.M.	2	3	4	5	6	7	8	9	10	11	12 Midnight
72°	74°	75°	75°	77°	75°	74°	70°	62°	55°	51°	48°

a. Find the amplitude of a sinusoidal function that models this temperature variation.

$$77 - 40 = 37 \quad 37/2 = 18.5$$

b. Find the vertical shift of a sinusoidal function that models this temperature variation.

$$\frac{77 + 40}{2} = 58.5$$

c. What is the period of a sinusoidal function that models this temperature variation?

1A.M. - 12A.M. 24 hrs

d. Use $t = 0$ at 5 P.M. to write a sinusoidal function that models this temp. Variation.

$$F(0) = T(5)$$

$$F(t) = 18.5 \sin \left(\frac{2\pi}{24} t \right) + 58.5$$

$$F(t) = 18.5 \sin \left(\frac{2\pi}{24} \times t - 5 \right) + 58.5$$

e. What is the model's temperature at 10 A.M.? Compare this to the actual value?

$$T(10) = 18.5 \sin \left(\frac{2\pi}{24} (10 - 5) \right) + 58.5$$

58.92 degrees Models temperature

52 degrees Actual Value

$$58.92 - 52 = 6.92$$

The models temperature is 6.92 degrees more than the actual value.

18. Find intersections and unions of the following given sets.

Ten students from a school appear in one or more subjects for an inter school quiz competition as shown in the table given below.

General Knowledge	Math	Science
Acel	Barek	Carlin
Acton	Bay	Acton
Anael	Max	Anael
Max	Kai	Kai
Carl	Anael	Dario
Dario	Carlin	Barek

Let G represents the set of students appearing for General Knowledge, M represents the set of students appearing for Math, and S represents the set of students appearing for Science.

Find $G \cap M$ and $G \cup S$

$$G \cup S = [Acel, Acton, Anael, Max, Carl, Dario] \cup [Carlin, Acton, Anael, Kai, Dario, Barek]$$

$$G \cap M = [Acel \times Barek] \quad [Carl \times Anael]$$

$$[Action \times Bay] \quad [Dario \times Carlin]$$

$$[Anael \times Max]$$

$$[Max \times Kai]$$

19. Heather invests \$4,900 in an account with a 3.5% interest rate, making no other deposits or withdrawals. What will Heather's account balance be after 5 years if the interest is compounded 2 times each year?

(SHOW WORK)

$$\text{Mvest} = \$4,900$$

$$\text{I R} = 3.5\%$$

Compound = 2 times yearly

$$\text{Fv} = \text{pv} \left(1 + \frac{r}{n}\right)^n$$

$$\text{Fv} = \$4,900 \left(1 + \frac{0.035}{2}\right)^2$$

$$\text{Fv} = \$4,900 \times (3.53\%)$$

So it will be

$$\text{Fv} = \$4,900 \times 0.0353 \times 5 \text{ years}$$

$$= \$864.85 \text{ in five years}$$

20. Find the area of the triangle with $a = 19$, $b = 14$, $c = 19$. Round to the nearest tenth.

(SHOW WORK)

$$\text{A} = 19 \quad \text{B} = 14 \quad \text{C} = 19$$

$$\text{A} = 123.64$$

$$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{S} = \frac{a+b+c}{2}$$

$$\text{A} = \frac{1}{4} \sqrt{-a^2 + 2(ab)^2 + 2(ac)^2 - b^4 + 2(bc)^2 - c^4}$$

$$= \frac{1}{4} \times \sqrt{-19^4 + 2 \times (19 \times 14)^2 + 2(19 \times 19)^2 - 14^4 + 2(14 \times 19)^2 - 19^4}$$

$$\text{Answer} = 123.64$$